Mode Coupling Prediction in Whispering Gallery Dielectric Resonators Modes

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***Abstract*— This letter describes a new method to locate the frequency band in which two whispering gallery modes (WGM's) are likely to be coupled. It is explained why this phenomenon is observed in single-crystal anisotropic dielectric resonators. This method is validated by computations of the resonant frequencies and the electromagnetic field cartography of WGM computed by finite-element method (FEM) for a sapphire resonator and experiments.**

I. INTRODUCTION

LOW-LOSS dielectric resonators operating on whispering gallery modes have recently found more applications in devices such as ultra-low-noise oscillators [1] and power combiners [2]. Whispering gallery mode (WGM) resonators in low-loss materials such as sapphire potentially offer very high Q since both radiation and conductors losses are very small in them [3]. In anisotropic materials, mode-coupling phenomena may occur between two mode families. The finite-element method (FEM) can be used successfully to compute resonant frequencies of different modes in those kinds of resonators [4] so the mode-coupling phenomenon can be computed with this software, but it is impossible to predict the frequency band at which it occurs. The method presented in this letter allows us to predict approximate frequency values for which the coupling takes place.

II. THEORY

The method is based upon the calculation of the relative position of caustic radii for different modes, which have similar frequencies. We first studied coupling between $WGH_{n,0,0}$ and $WGE_{n,0,1}$ modes and between $WGE_{n,0,0}$ and $WGH_{n,0,1}$ modes. The permittivity parallel and perpendicular to the z axis of a cylindrical anisotropic crystal are defined as ε_z and ε_ϕ, respectively. We use the optical beam theory [5], [6] to determine the caustic radius of two modes considered.

Applying this theory, we can find radial wave β_r number as

$$\beta_r^2 = k^2 - \beta_z^2 - \beta_\theta^2 \tag{1}$$

with

$$k^2 = \varepsilon_i \frac{\omega^2}{c^2}$$

where $\varepsilon_i = \varepsilon_z$ or $\varepsilon_i = \varepsilon_\phi$ is the dielectric constant for WGH and WGE modes, respectively, k is the dielectric wave number, and c is the speed of the light in vacuum. β_z and β_θ are, respectively, the axial and azimuthal wave numbers. β_θ and β_r depend on the azimuthal number n of the mode according to the following approximate relationships:

$$\beta_\theta^2 \approx \frac{n^2}{r^2}$$
$$\beta_r^2 \approx \varepsilon_i \frac{\omega^2}{c^2} - \beta_z^2 - \frac{n^2}{r^2}. \tag{2}$$

If $\beta_r > 0$, the caustic radii a_{cE} and a_{cH} for the WGE and WGH modes can be found from the following formula [6]:

$$a_{cE}^2 = \frac{n^2}{\varepsilon_\phi \frac{\omega^2}{c^2} - \beta_{zE}^2}$$
$$a_{cH}^2 = \frac{n^2}{\varepsilon_z \frac{\omega^2}{c^2} - \beta_{zH}^2}. \tag{3}$$

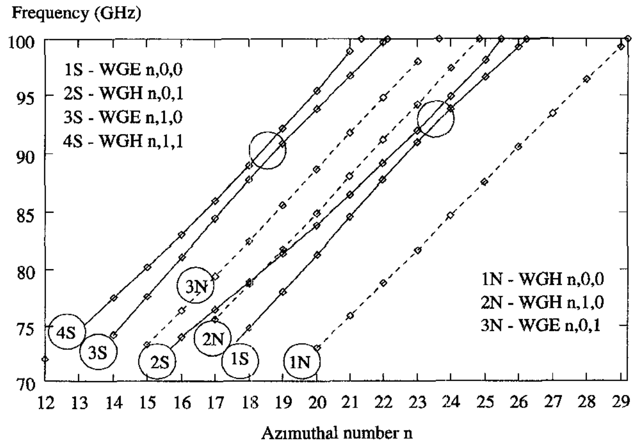

Fig. 1. Experimental resonant frequencies versus azimuthal mode number for the symmetric (1S ··· 4S) and antisymmetric modes (1N ··· 3N) of a single-crystal sapphire resonator having d_r = 9.96 mm and h_r = 2.093 mm.

The approximate frequency value for which the mode coupling takes place can be evaluated from equality of the caustic radii a_{cE} and a_{cH} of the WGE and WGH mode families.

Since the axial propagation constants satisfy the relationships (4)

$$\beta_{zH} = \frac{\delta_H \pi}{h}$$
$$\beta_{zE} = \frac{\delta_E \pi}{h} \tag{4}$$

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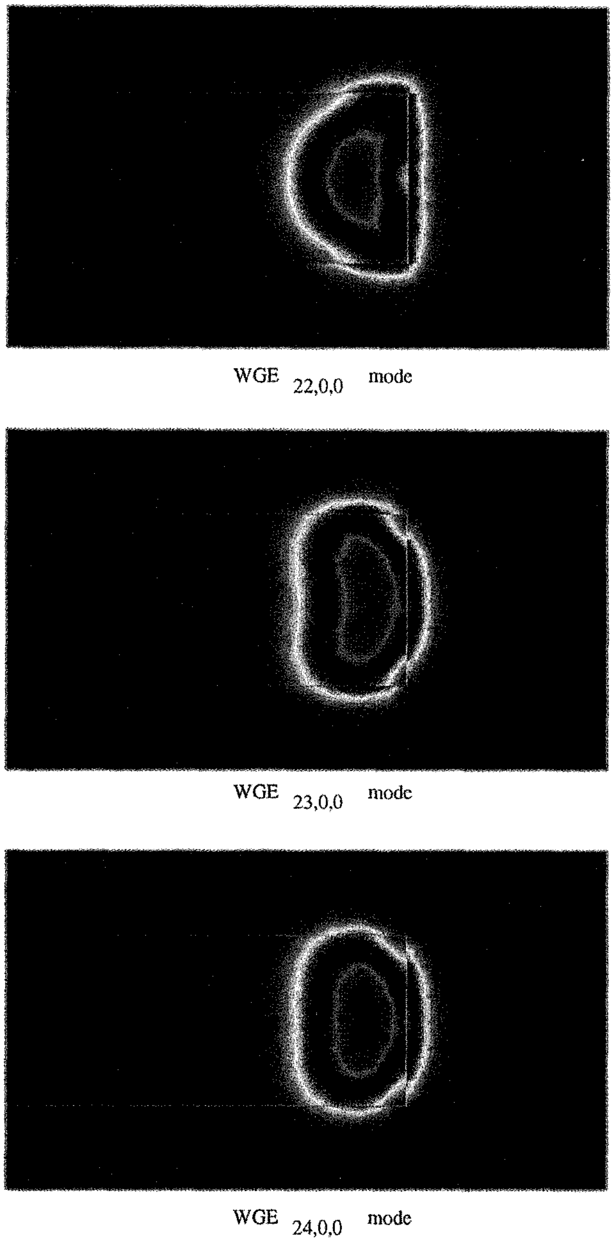

Fig. 2. Computed plots of the electric field energy density of $\mathrm{WGE}_{n,0,0}$ modes for $n = 22$–23–24.

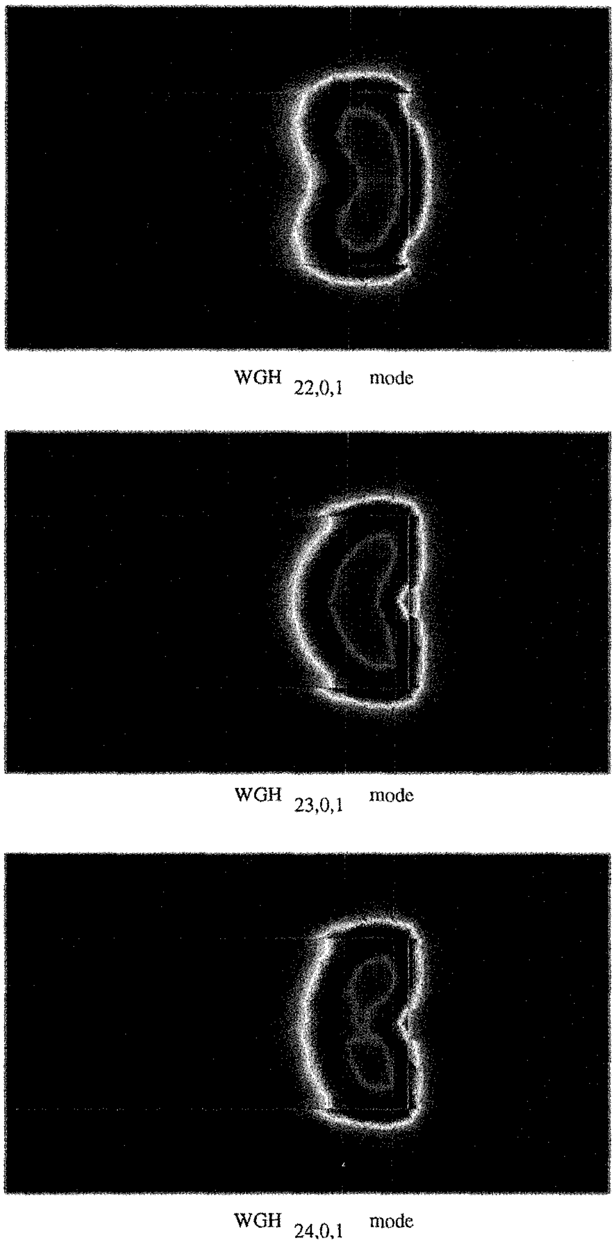

Fig. 3. Computed plots of the electric field energy density of $\mathrm{WGH}_{n,0,1}$ modes for $n = 22$–23–24.

then the coupling frequency value f_{EH} is given by

$$f_{EH} = \frac{c}{2h}\left[\frac{\delta_H^2 - \delta_E^2}{\varepsilon_z - \varepsilon_\phi}\right]^{1/2} \tag{5}$$

where δ_H and δ_E represents a fractional number of variations for the electromagnetic field in the axial direction for the WGH and WGE modes, respectively, and h is the height of the resonator. If we consider material with $\varepsilon_z > \varepsilon_\phi$, like sapphire, then f_{EH} can be real only if $\delta_H^2 > \delta_E^2$. It means that the WGE modes are coupled with WGH having higher axial number. For the particular case when $\delta_E \approx 1$ and $\delta_H \approx 2$ we obtain from (5):

$$f_{EH} \approx \frac{260}{h\,(\mathrm{mm})\,\sqrt{\varepsilon_z - \varepsilon_\phi}}\ \mathrm{GHz}. \tag{6}$$

From (5) or (6) we can find out that the approximate value of frequency at which mode coupling appears depends mainly on the height of the resonator and on its anisotropy.

III. Application to Sapphire Resonators

We investigated WG modes in a sapphire resonator at W band using a Hewlett Packard 8510 network analyzer. We used a sapphire with $d_r = 9.96$ mm in diameter and $h_r = 2.096$ mm in height. The permittivity has proved to be $\varepsilon_z = 11.35$ and $\varepsilon_\phi = 9.37$, evaluated by the method described in [7]. For this experiment, DR is excited by two microstrip lines and set on a low permittivity support. In these conditions, the coupling system does not disturb DR modes.

The experimental curves showing the resonant frequencies versus azimuthal number for the first four symmetric modes 1S $\cdots$ 4S and the three antisymmetric modes 1N $\cdots$ 3N are presented in Fig. 1. We obtain coupling phenomenon in a frequency band from 88–95 GHz. The coupling frequency given by (6) is 88.15 GHz. We can see that the frequency ranges of coupling phenomenon for $WGE_{n,0,0}$ (1S) and $WGH_{n,0,1}$ (2S) modes and for $WGE_{n,1,0}$ (3S) and $WGH_{n,1,1}$ (4S) modes are quite the same, indicating that the radial number has only inferior influence on this phenomenon. On the other hand, $WGH_{n,0,0}$ (1N) and $WGE_{n,0,1}$ (3N) are not coupled.

We used the FEM in free oscillations to study the electric field density of the $WGE_{n,0,0}$ and $WGH_{n,0,1}$ mode families. For this simulation, the dielectric resonator is placed in a metallic cavity. Figs. 2 and 3 show the electric field energy density respectively of the $WGE_{n,0,0}$ modes and $WGH_{n,0,1}$ modes for azimuthal number under and above coupling frequency ($n = 22$–23–24). The electric field relative to WGE modes moves from the inner region of the resonator to its outer region. The opposite electric field transformation may be observed for $WGH_{n,0,1}$ modes.

Therefore, the phenomenon we have investigated may be considered a mode transformation that takes place near the frequency determined by (5) and (6).

IV. Conclusion

- It has been shown that in anisotropic dielectric resonators, the $WGE_{n,m,p}$ modes can be coupled to $WGH_{n,m,p+1}$ modes family for high azimuthal mode numbers.
- The frequency at which coupling occurs may be easily predicted from a simple formula.
- A rigorous study using FEM can characterize accurately the electromagnetic field distribution and the resonant frequencies for the modes involved in coupling phenomenon.

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